The Big Idea

The development of devices to measure time like a pendulum led to the analysis of *periodic motion*. Motion that *repeats itself in equal intervals of time* is called *harmonic motion*. When an object moves back and forth over the *same path* in harmonic motion it is said to be *oscillating*. If the amount of motion of an oscillating object (the distance the object travels) stays the same during the period of motion, it is called *simple* harmonic motion (SHM). A grandfather clock's pendulum and the quartz crystal in a modern watch are examples of SHM.

Key Concepts

*   The oscillating object does not lose any energy in SHM. Friction is assumed to be zero.
*   In harmonic motion there is always a *restorative force*, which acts in the opposite direction of the displacement. The restorative force changes during oscillation and depends on the position of the object. In a spring the force is given by Hooke's Law, -kx; in a pendulum it is the component of gravity along the path, or directly opposite that of the displacement.
*   Objects in simple harmonic motion do not obey the "Big Three" equations of motion because the acceleration is not constant. As a spring compresses, the force (and hence acceleration) increases. As a pendulum swings, the tangential component of the force of gravity changes, so the acceleration changes.
*   The period, T, is the amount of time for the harmonic motion to repeat itself, or for the object to go one full cycle. In SHM, T is the time it takes the object to return to its exact starting point and starting direction.
*   The frequency, *f*, is the number of cycles an object goes through in 1 second. Frequency is measured in Hertz (Hz). 1 Hz = 1 cycle per sec.
*   The amplitude, A, is the distance from the *equilibrium* (or center) *point* of motion to either its lowest or highest point (*end points*). The amplitude, therefore, is half of the total distance covered by the oscillating object. The amplitude can vary in harmonic motion but is constant in SHM.
*   The kinetic energy and the speed are at a maximum at the equilibrium point, but the potential energy and restorative force is zero there.
*   At the *end points* the potential energy is at a maximum, while the kinetic energy and speed are zero. At the end points the restorative force and acceleration are at a maximum.
*   In SHM since energy is conserved the most fruitful method of calculating position and velocity is to set the total energy equal to the sum of kinetic and potential energies. In most problems this will be far easier than using the "Big Two". Similarly force and acceleration are best calculated by using $\Sigma F = ma$.

Key Equations

- $T = 1/f$; Period and frequency are inversely related.

- $T_{sp} = 2\pi \sqrt{\dfrac{m}{k}}$; the period of oscillation in seconds for a mass oscillating on a spring depends on the mass of the object on the spring and the spring constant.

- $T_p = 2\pi \sqrt{\dfrac{L}{g}}$; the period of oscillation in seconds for a pendulum (i.e. a mass swinging on a string) swinging at small angles ($\theta < 15°$) with respect to the vertical depends on the length of the pendulum and the acceleration due to gravity.

- $x(t) = x_0 + A \cos[2\pi f(t - t_0)]$; equation for the position of an object in SHM.

- $v(t) = -2\pi f A \sin[2\pi f(t - t_0)]$; equation for the velocity of an object in SHM.

Simple Harmonic Motion Problem Set

1. While treading water, you notice a buoy way out towards the horizon. The buoy is bobbing up and down in simple harmonic motion. You only see the buoy at the most upward part of its cycle. You see the buoy appear 10 times over the course of one minute.

   a. What is the force that is leading to simple harmonic motion?
   b. What are the period *(T)* and frequency *(f)* of its cycle? Use the proper units.

2. A rope can be considered as a spring with a very high spring constant *k*, so high, in fact, that you don't notice the rope stretch at all before it "pulls back."

   a. What is the *k* of a rope that stretches by 1 mm when a 100 kg weight hangs from it?
   b. If a boy of 50 kg hangs from the rope, how far will it stretch?
   c. If the boy kicks himself up a bit, and then is bouncing up and down ever so slightly, what is his frequency of oscillation? Would he notice this oscillation? If so how? If not, why not?

3. If a 5.0 kg mass attached to a spring oscillates 4.0 times every second, what is the spring constant *k* of the spring?

4. A horizontal spring attached to the wall is attached to a block of wood on the other end. All this is sitting on a frictionless surface. The spring is compressed 0.3 m. Due to the compression there is 5.0 J of energy stored in the spring. The spring is then released. The block of wood experiences a maximum speed of 25 m/s.

   a. Find the value of the spring constant.
   b. Find the mass of the block of wood.
   c. What is the equation that describes the position of the mass?
   d. What is the equation that describes the speed of the mass?
   e. Draw three complete cycles of the block's oscillatory motion on an *x* vs. *t* graph.

5. Give some everyday examples of simple harmonic motion.

6. Why doesn't the period of a pendulum depend on the mass of the pendulum weight? Shouldn't a heavier weight feel a stronger force of gravity?

7. The pitch of a Middle C note on a piano is 263 Hz. This means when you hear this note, the hairs in your ears wiggle back and forth at this frequency.

   a. What is the period of oscillation for your ear hairs?
   b. What is the period of oscillation of the struck wire within the piano?
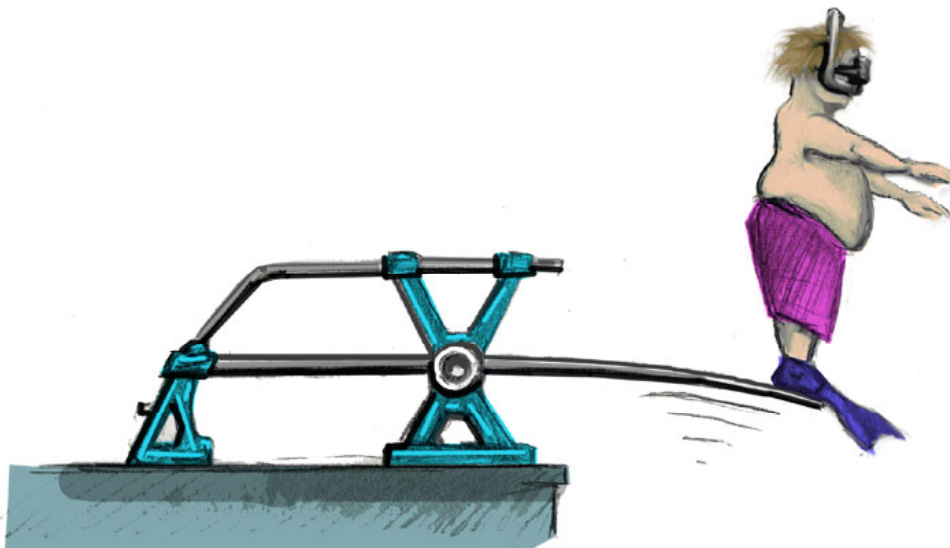
8. The effective k of the diving board shown here is 800 N/m. (We say effective because it bends in the direction of motion instead of stretching like a spring, but otherwise behaves the same.) A pudgy diver is bouncing up and down at the end of the diving board, as shown. The y vs t graph is shown below.
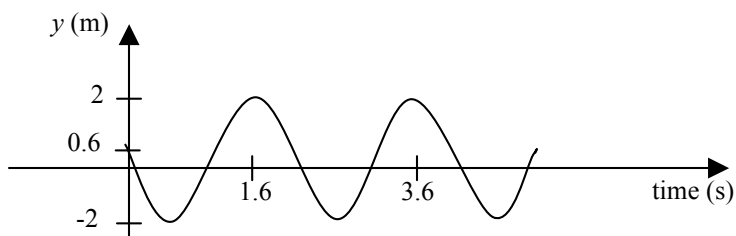
a. What is the distance between the lowest and highest points of oscillation?
b. What is the y-position of the diver at times $t = 0$ s, $t = 2$ s, and $t = 4.6$ s?
c. Estimate the man's period of oscillation.
d. What is the diver's mass?
e. Write the sinusoidal equation of motion for the diver.

Sunspot Cycles 1749 – July 2003
Average Monthly Count
source: International Sunspot Count (Brussels SIDC Index)
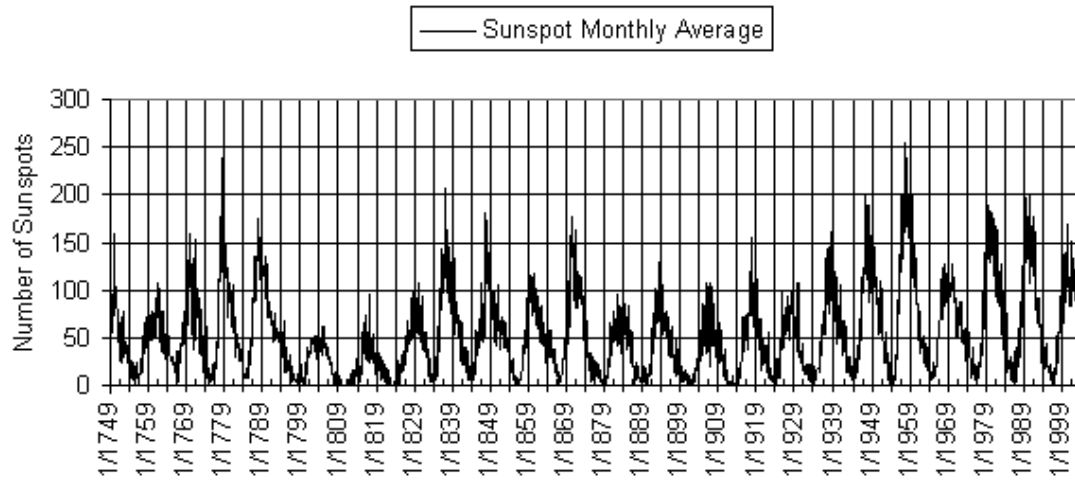
9. The Sun tends to have dark, Earth-sized spots on its surface due to kinks in its magnetic field. The number of visible spots varies over the course of years. Use the graph of the sunspot cycle above to answer the following questions. (Note that this is real data from our sun, so it doesn't look like a *perfect* sine wave. What you need to do is estimate the *best* sine wave that fits this data.)

    a. Estimate the period T in years.
    b. When do we expect the next "solar maximum?"

10. The pendulum of a small clock is 1.553 cm long. How many times does it go back and forth before the second hand goes forward one second?

11. On the moon, how long must a pendulum be if the period of one cycle is one second? The acceleration of gravity on the moon is 1/6 th that of Earth.

12. A spider of 0.5 g walks to the middle of her web. The web sinks by 1.0 mm due to her weight. You may assume the mass of the web is negligible.

    a. If a small burst of wind sets her in motion, with what frequency will she oscillate?
    b. How many times will she go up and down in one s? In 20 s?
    c. How long is each cycle?
    d. Draw the *x* vs *t* graph of three cycles, assuming the spider is at its highest point in the cycle at t = 0 s.

13. A mass on a spring on a frictionless horizontal surface undergoes SHM. The spring constant is 550 N/m and the mass is 0.400 kg. The initial amplitude is 0.300 m.

      a.   At the point of release find:
          i. the potential energy
          ii. the horizontal force on the mass
          iii. the acceleration as it is released
      b. As the mass reaches the equilibrium point find:
          i. the speed of the mass
          ii. the horizontal force on the mass
          iii. the acceleration of the mass
      c. At a point .150 m from the equilibrium point find:
          i. the potential and kinetic energy
          ii. the speed of the mass
          iii. the force on the mass
          iv. the acceleration of the mass
      d. Find the period and frequency of the harmonic motion.

14. A pendulum with a string of 0.750 m and a mass of 0.250 kg is given an initial amplitude by pulling it upward until it is at a height of 0.100 m more than when it hung vertically. This is point P. When it is allowed to swing it passes through point Q at a height of .050 m above the equilibrium position, the latter of which is called point R.

      a.   Draw a diagram of this pendulum motion and at points P, Q, and R draw velocity and acceleration vectors. If they are zero state that also.
      b.   At point P calculate the potential energy.
      c.   At point R calculate the speed of the mass
      d.   At point Q calculate the speed of the mass
      e.   If the string were to break at points P, Q and R draw the path the mass would take until it hit ground for each point.
      f.   Find the tension in the string at point P.
      g.   Find the tension in the string at point R
      h.   Find the period of harmonic motion.